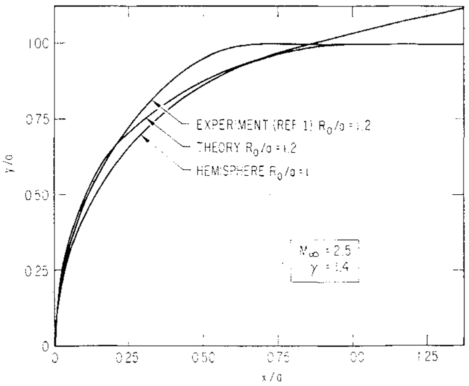

Fig. 2 Comparison of theoretical shape with experimental shape from Ref. 1.

where a is a reference length. For the hemisphere-cylinder considered in this note, a is the radius of the circular cylinder. With the aid of Eq. (11), integration of Eq. (9) yields

$$\frac{x}{a} = \left(\frac{T^2}{R_0}\right)\left(\frac{R_0}{a}\right)\left(\frac{f \tan\alpha}{\cos\alpha} - \int_0^\alpha \frac{f}{\cos^3\alpha}\, d\alpha\right) \tag{12}$$

Once the functions f and R_0/T^2 are known, the equilibrium shape can be computed from Eqs. (11) and (12) for particular values of R_0/a. For the shape given in this note, f was computed from Eq. (6) with the Newtonian pressure relation

$$p_1/p_0 = \cos^2\alpha + (\gamma M_\infty)^{-1} \sin^2\alpha \tag{13}$$

and the velocity relation

$$u_1/u_\infty = \{([2/(\gamma - 1)M_\infty^2] + 1)[1 - (p_1/p_\infty)^{(\bar{\gamma} - 1)/\bar{\gamma}}]\}^{1/2} \tag{14}$$

Equation (14) corresponds to isentropic expansion from the stagnation point streamline to the local pressure p_1. γ is the freestream specific heat ratio, $\bar{\gamma}$ is the value of γ behind the bow shock, and M_∞ is the freestream Mach number. With the aid of Eqs. (13) and (14), Eq. (7) can be written as

$$R_0/T^2 = (\{[2/(\gamma - 1)M_\infty^2] + 1\}[(\bar{\gamma} - 1)/\bar{\gamma}][1 - 1/(\gamma M_\infty^2)])^{1/2} \tag{15}$$

A comparison of the theoretical shapes obtained from Eqs. (10) and (11) and a typical experimental shape is shown in Fig. 1. Hemisphere-cylinder models with $a = \frac{1}{2}$ in. and overall length of 6 in. were constructed from pure teflon and teflon mixed with 15% graphite. Tests were conducted in the Aerospace Corp. arc tunnel with $M_\infty = 4.5$, $p_0 = 0.2$ atm, and stagnation temperature = 4000°K. Motion pictures of the erosion process revealed that both models approached the same blunted shape, which remained unchanged with further recession. Additional tests with models of different sizes are required to ascertain if this shape is unique.

The theoretical shapes shown in Fig. 1, which are based on the Newtonian pressure formula, terminate at $\alpha = 54°$ because the quantity f in Eq. (11) reaches a maximum. Because further increase in α reduces the values of both x/a and y/a, a branch curve is formed which intersects the y/a axis above the origin. This branch has no physical meaning. The other branch which starts at the stagnation point could probably be continued around the corner if Eq. (1) were modified by the addition of a constant term to the right-hand side, and if Eq. (13) were replaced by a better approximation in the shoulder region.

Two values of R_0/a were used to plot the theoretical shape. Value 1.42 was measured from the models, and the value 1.66 was chosen so that the maximum ordinate of the profile reached the diameter of the cylinder. The former curve fits the experimental shape very well, while the latter is more blunt. Further experimental investigations will be required to ascertain the generality of $R_0/a = 1.42$ for various size bodies under the same flow conditions.

If the theoretical shape extended to $\alpha = \pi/2$, the value of R_0/a might be determined by the condition $y = a$ at $\alpha = \pi/2$. By raising the pressure over the value given by Eq. (13) when $\alpha > 50°$, it was possible to extend the present analysis to larger values of α, but extension completely around the corner would still require a modification of Eq. (1) because its present form is incorrect as $\alpha \rightarrow \pi/2$.

Figure 2 shows a comparison of the theory with the experimental shape obtained by Simpkins at $M = 2.5$. Value $R_0/a = 1.2$ was measured from a photograph of the model, and the theoretical curve was plotted with the same value of R_0/a. For this lower Mach number, quantity f does not reach a maximum; therefore the curve does not terminate as in Fig. 1. The theory agrees well with the experimental shape near the nose, but deviates from the actual shape in the shoulder region.

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A Shock Wave Attenuation Treatment for Ballistic Ranges

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As is well known, the ballistic range consists basically of a long (usually cylindrical) evacuated tank, instrumented at various stations along its length. Projectiles are gun-launched along a path parallel to the axis of the tank and flown past the various instrumented stations. The Defense Research Establishment Valcartier (DREV) has two ballistic range facilities in current operation: Range 3 has a medium size 6-ft-diam tank and Range 5 is a "large" facility,

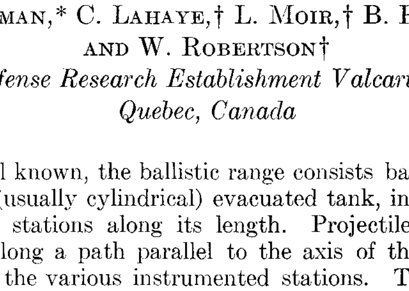

Fig. 1 Wake centerline displacement measured from schlieren movies of a 2.7-in. sphere firing in the asymmetric geometry of Range 5. (Mach number was 13.5 and the ratio of range diameter to sphere diameter was 44.)

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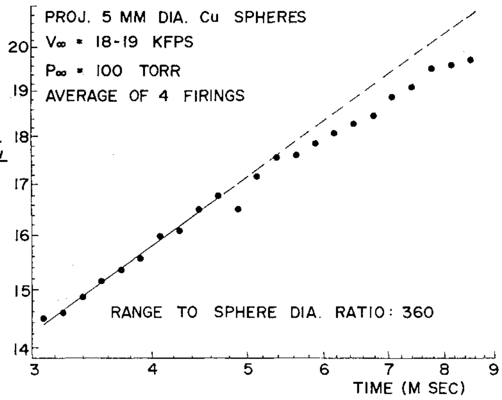

Fig. 2 Schlieren measurements of the growth of the wake diameter D_w (normalized to projectile diameter D) show the temporary freezing of wake growth occurring at the time corresponding to the return of the reflected shock system; the change in wake diameter is about 0.75 body diam or 5%. (Mach number was about 17 and ratio of range diameter to sphere diameter was 360.)

10 ft in diameter. Both facilities are devoted to a joint DREV-ARPA program which has developed around four basic experiments devised for making spatially resolved or point measurements of mass density, temperature, velocity, and charge density in turbulent hypersonic wakes.[1]

Effort at DREV has recently been devoted to finding a means to alleviate the effects of the reflection at the walls of the range tank of the bow and wake shock system originating with a hypersonic projectile. This reflected shock system returns to the center of the range where it impinges on the wake of the projectile. The effect of this shock wave perturbation on the wake has been seen in all of the point-measurement experiments at DREV. The total result of the disturbance on the wake has not been completely elucidated, but some of the major effects have been explained. One of these is the physical displacement of the wake by any asymmetry in the reflected shock wave system, causing, for example, loss of information as to radial position in the wake. Such asymmetry could be produced by some object in the range such as a Schlieren mirror, a walkway, or simply by firing a projectile slightly off-axis.

Figure 1 shows a cross section drawing of the Range 5 tank. Indicated on the figure is the asymmetric reflecting shock wave system. The bottom part of the shock wave reflecting from the walkway is tending to diverge, at least initially, and impinges on the wake early, since the highest portion of the walkway is about 11 in. closer to the center of the range than the range wall. The top portion of the shock system is converging, but since it has to travel further, it takes more time before it arrives back into the center of the range. Also shown is actual data obtained from Hycam Schlieren movie studies of the centerline motion of the wakes of 2.7-in.-diam hypersonic spheres launched on Range 5. First the wake is driven up by the velocity field induced by the passage of that part of the shock system originating at the walkway; after a certain delay the wake is driven down again by the portion of the shock system coming from the roof of the range. This part of the wave is strongly focusing, and in addition to displacing the wake, causes a pinching.

The effect of shock reflections on point measurements using hot wires has previously been reported by Fox et al.[2] for the case of relatively low (4000 fps) velocity projectiles in a small range-like facility. Work has been done at Lincoln Laboratory[3] concerning the effect of reflected shocks on integrated electron (line) density measurements using a cavity of cylindrical geometry, but no effect attributable to shock reflections was observed, even though observations were made with cavities of various diameters. These contradictory results do indicate a degree of sensitivity of detection of the effects of shock wave perturbation which is dependent on whether the measurement is of the "point" or "integrated" type. Investigations at DREV based on a few very small sphere firings in Range 3 also indicate the existence of probable reflected shock effects (Fig. 2), despite significant differences in the velocity and the ratio of range diameter to projectile diameter as compared to those employed in Range 5.

The effect of the reflected shock system on a point measurement can best be seen under low-pressure conditions when the wake is laminar, since in such wakes the reflected shock signature is not hidden by turbulent fluctuations. Figure 3 shows data from a typical signal from an electrostatic probe in the laminar wake of a 2.7-in. sphere firing at 15,000 fps and 1 torr in nitrogen on Range 5. Also shown for comparison is a predicted probe current obtained by assuming for simplicity a nondeformed, nondecaying gaussian distribution of charge density of appropriate width and symmetrical about the wake centerline, and displacing this pattern of charge density about a hypothetical probe as would be prescribed by the wake centerline motion shown in Fig. 1. The agreement between the experimental behavior and the predicted probe current is seen to be reasonably good. Essentially one concludes that a major effect of the return of an asymmetrical shock system on the wake is the result of a physical displacement of the wake, at least at low pressures. The results imply that comparatively little change in electron density has resulted from passage of the shock system through the wake and this is consistent with the observation at Lincoln Laboratory of no apparent effect on line densities observed in cylindrical cavity measurements.

The studies at DREV have been concerned with a wide range of shock deflection schemes and attenuation schemes

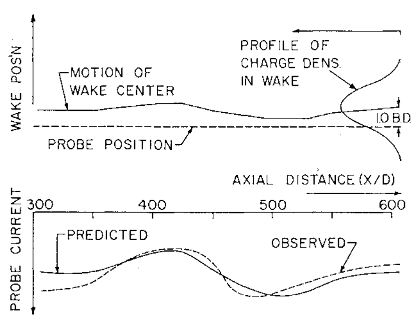

Fig. 3 The reflected shock signature seen by an electrostatic probe is due to displacement of the ionization profile with respect to the probe by the velocity fields associated with the shocks.

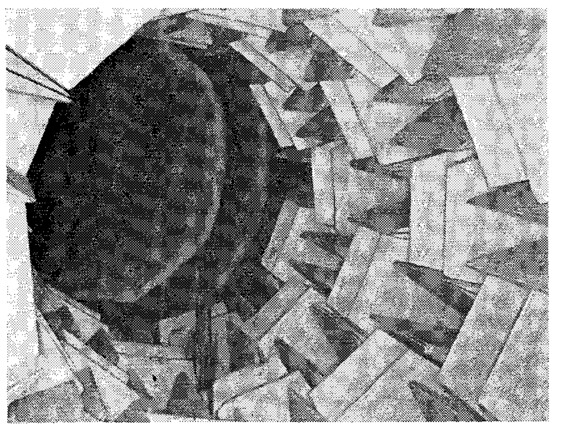

Fig. 4 Anechoic fiberglas wedge treatment installed in a scaled cylinder for test and evaluation; the planes of symmetry through the individual wedges were approximately normal to the impacting shock wave; electrostatic probes are also visible in the cylinder.

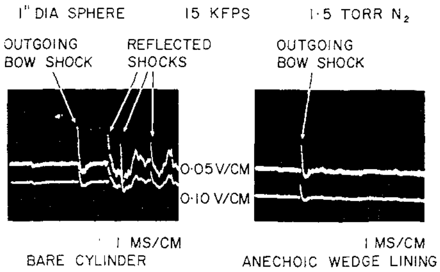

Fig. 5 Pressure transducer measurements in the anechoic wedge treatment compared to results in a bare cylinder; the reflected shocks are not detected in the treated cylinder.

using acoustic materials. Deflection schemes have not proved adequate because of diffraction effects, while normally highly absorbent acoustic materials appear very reflective at the low pressures to which the ranges are evacuated. However, a treatment based on smoothly tapered fiberglas wedges, installed as shown in Fig. 4, has been found to produce significant improvement. Figure 5 compares test results obtained in firings through untreated and treated cylinders using Atlantic Research Corporation LC-5 pressure transducers; the reflected shocks evident in the former case cannot be detected in the latter. Electrostatic probe results indicate a significant reduction in wake disturbance with the wedge treatment, provided the range geometry is symmetrical. Some residual effect does remain, probably due to the presence of a smeared pressure distribution persisting as a result of inefficient energy absorption by the treatment at low ambient pressures.

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Critical Height Phenomenon for Vertical Jets Mounted in Flat Surfaces

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Introduction

An investigation was made into the effects of the flow between two flat, parallel surfaces where the flow originated from a nozzle in one of the surfaces exhausting normal to the other surface. One surface, which we shall call the vehicle for convenience, was finite in area and cantilevered on the air supply line. The other surface, which we shall call the ground plane, was relatively infinite but adjustable for the purposes of the investigation. The configuration is that of an air-bearing type ground effect machine. The flow through the nozzle acted as a source of thrust for the vehicle portion of the test apparatus. At large distances between the two surfaces the presence of the ground plane had no effect on the thrust experienced by the vehicle. As the two surfaces were brought closer together and the flow became turned by the ground plane so that a velocity parallel to the surfaces was created, the amount of net thrust experienced by the vehicle decreased. At a certain distance, herein called the critical height, the net thrust acting on the vehicle was zero. For separation of the vehicle and ground plane by less than this distance the net thrust became negative, or toward the ground plane. It was found that, of the parameters tested, the value of the critical height depended only on the area of the vehicle surface. This study expands upon results obtained by Spreeman and Sherman[1] for similar configurations and by von Glahn[2] for nozzles with no surfaces around them. Different parameters were varied and some parameters were varied over wider ranges than in the previous studies. Data obtained in the experiment were nondimensionalized in a different manner resulting in the discovery of what we shall call the critical height phenomenon.

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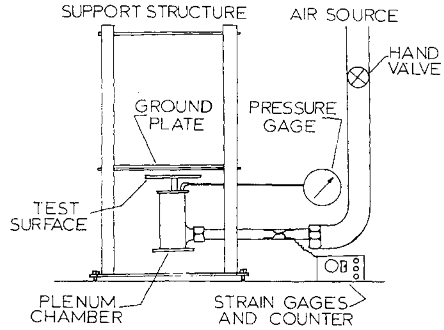

Fig. 1 Test apparatus.

Test Apparatus and Procedure

A simple cold-flow apparatus was used with air as the working fluid (Fig. 1). The apparatus was inverted with respect to the normal aircraft-ground orientation for convenience in taking the required data and making the necessary variations of physical parameters. The flow was provided from a high-pressure compressed air source at the Auburn University High Speed Wind Tunnel facility. The vehicle surfaces tested consisted of circular plates $2\frac{3}{8}$, 7, $9\frac{1}{4}$, and 11 in. in diameter and one elliptical planform approximately 5×10 in. The elliptical planform had the same area as the 7-in. circle and was used to determine the effect of small changes in the symmetry of the vehicle surface. The aspect ratio of the ellipse was less than three. Aspect ratio as used here is the length of the major axis squared divided by the area of the surface. In this respect the aspect ratio of the circular planforms was $4/\pi$. Three different nozzles were used: a 1-in. diam straight-walled nozzle; a $\frac{1}{2}$-in. diam straight-walled nozzle; and a $\frac{1}{2}$-in. diam nozzle which diverged to $\frac{5}{8}$ in. with a total angle of 24°. The ground plane was a heavy metal plate that was not moved by the flow. Deflection of the vehicle surface due to the net thrust acting upon it was not measurable. The separation between the two surfaces was varied by adjusting the height of the ground plane. The thrust experienced by the vehicle was measured using four electrical strain gages in a Wheatstone bridge arrangement. The strain gages were mounted on the air supply line which also served as the sole support for the vehicle portion of the apparatus.

A large number of tests were made using all possible combinations of planforms and nozzles at various mass flows and heights. The pressure in the plenum chamber upstream of the nozzle was varied to provide both choked and unchoked flow to the nozzle. In all cases, the thrust produced in the